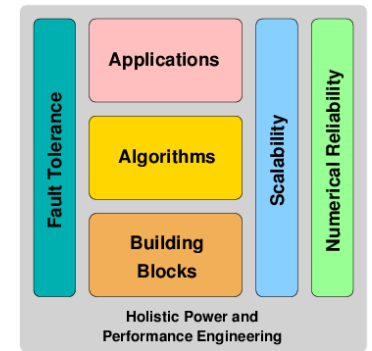


ESSEX II – Annual Report 2016

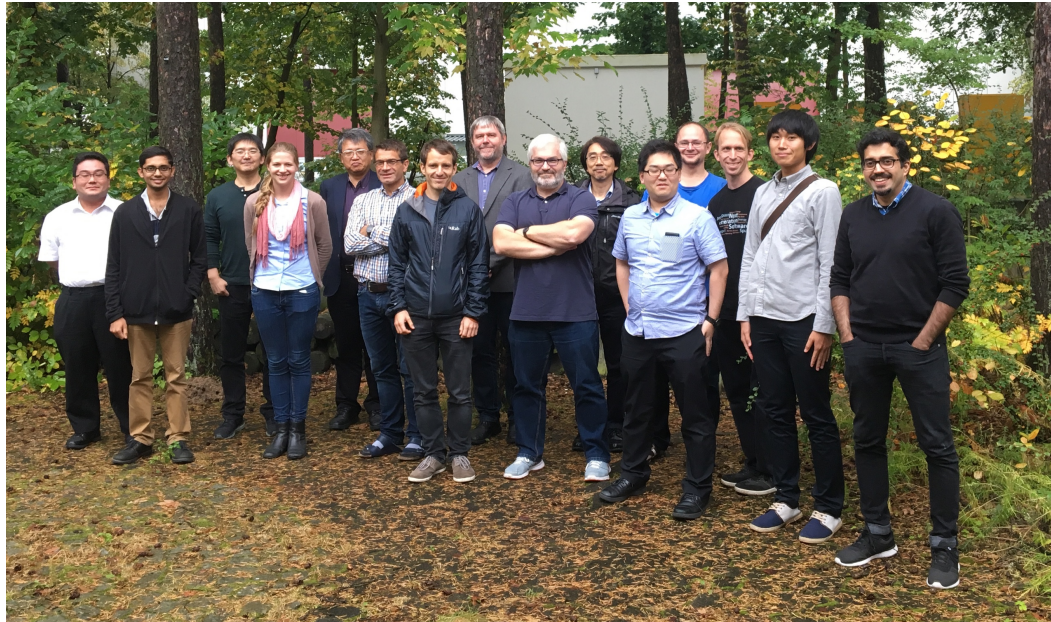
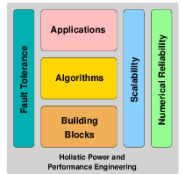


Gerhard Wellein
Bruno Lang
Achim Basermann
Holger Fehske
Georg Hager
Tetsuya Sakurai
Kengo Nakajima

Computer Science, University Erlangen
Applied Computer Science, University Wuppertal
Simulation & SW Technology, German Aerospace
Institute for Physics, University Greifswald
Erlangen Regional Computing Center
Applied Mathematics, University of Tsukuba
Computer Science, University of Tokyo

Garching, March 20th, 2017

ESSEX-II team



21 (accepted) publications

21 Talks

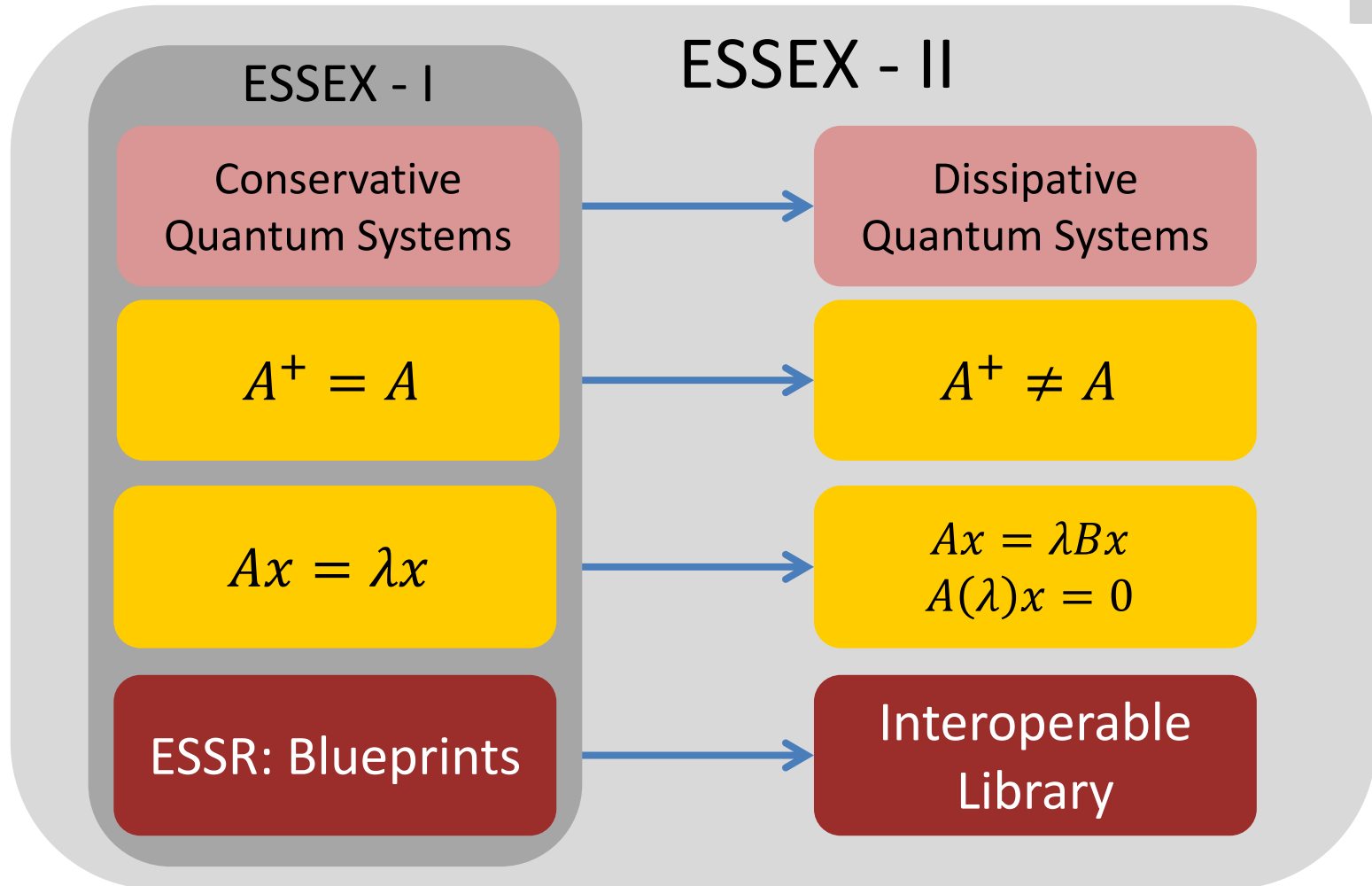
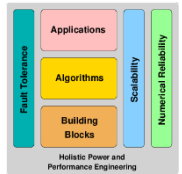
Full project in person meetings:

- January 21 / 22 (Berlin): Start-up Meeting
- September 19 – 22 (Erlangen): Coding week

Continuous meetings / visits / video conferences

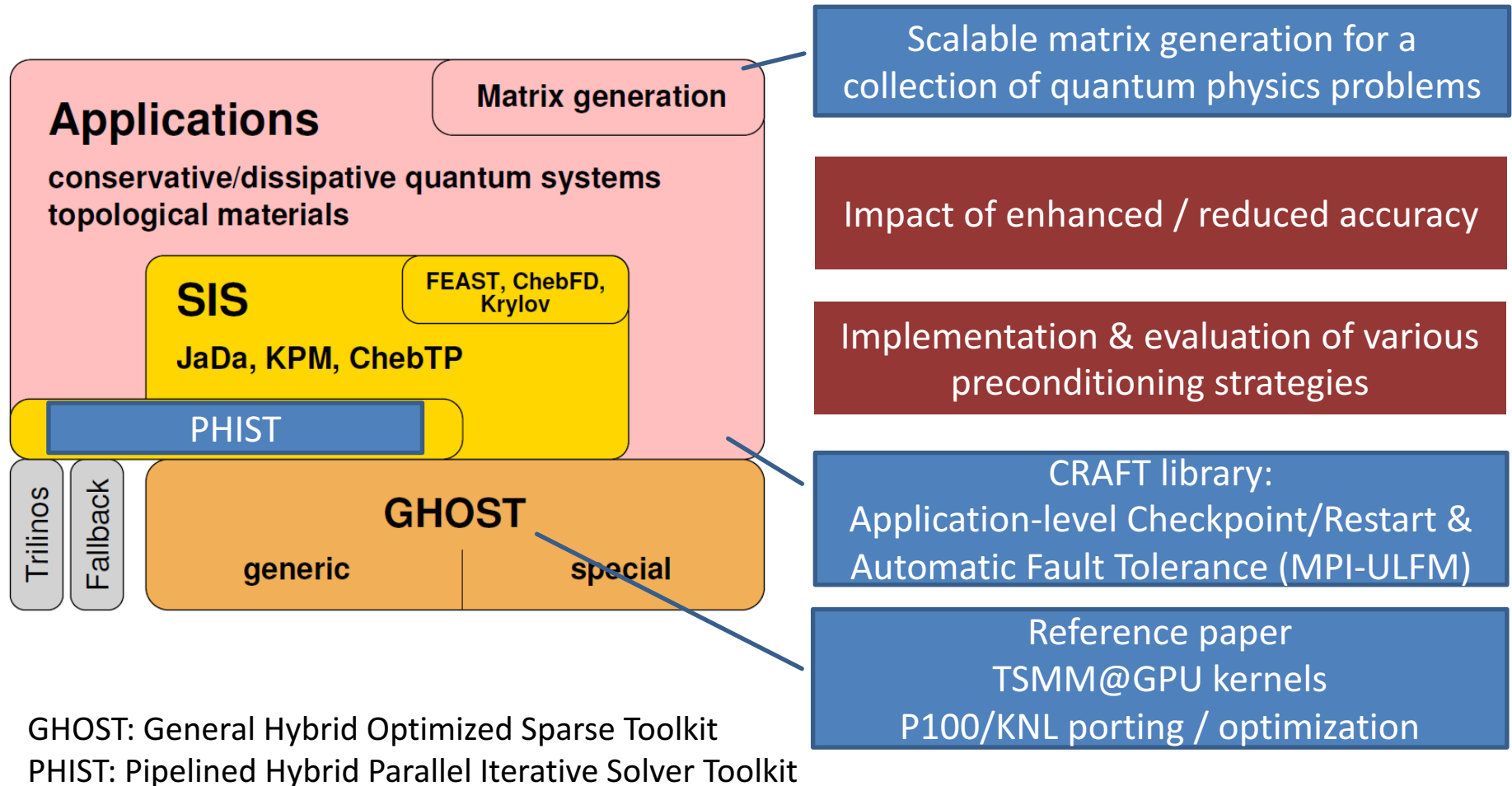
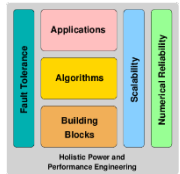
Joint PMAA 2016 minisymposium and ISC 2017 minisymposium

Motivated by quantum physics applications



→ Sparse eigenvalue solvers of broad applicability

ESSEX-II: Focus topics in 2016

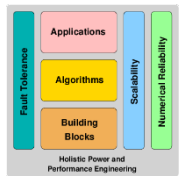


ESSEX project webpage: <http://blogs.fau.de/essex/>

Solvers & Hardware efficiency

Hardware efficient (linear) solvers

Kaczmark solver (used in BEAST-C)



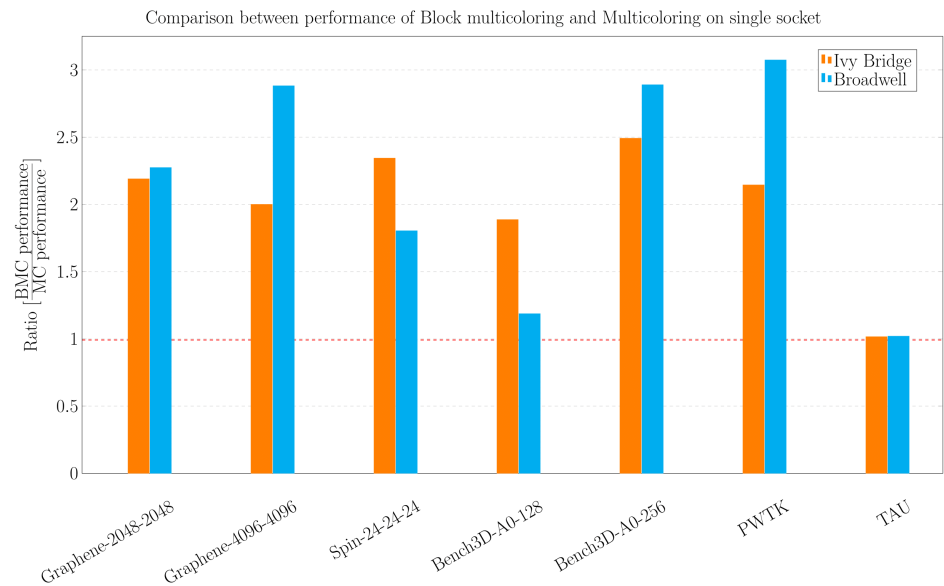
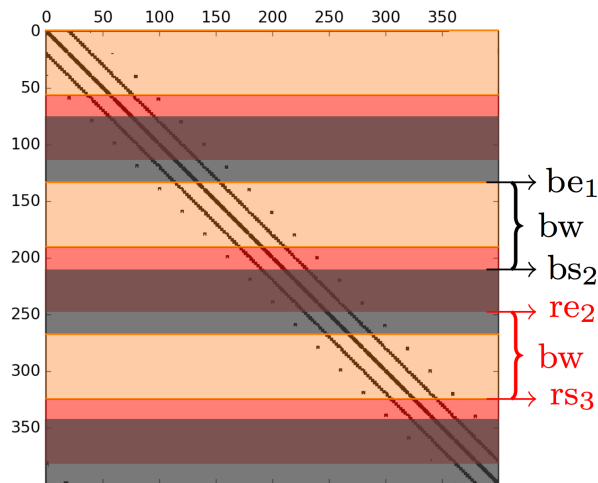
```

for row = 0 : nrows do
  scale = b[row]
  norm = 0
  for idx = rowptr[row] : rowptr[row + 1] do
    scale- = val[idx] * x[col[idx]]
    norm+ = val[idx] * val[idx]
  end for
  scale* = omega/norm
  for idx = rowptr[row] : rowptr[row + 1] do
    x[col[idx]] += scale * val[idx]
  end for
end for
    
```

$$x^{k+1} = x^k + \omega * \frac{(b_i - \langle A_i, x^k \rangle)}{\|A_i\|^2} * A_i^T$$

Parallelization

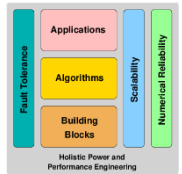
- Standard: D2-multicoloring (distance: 2) (D2-MC)
- **New: block-multicoloring (BMC)**
(+ D2-MC, where needed)



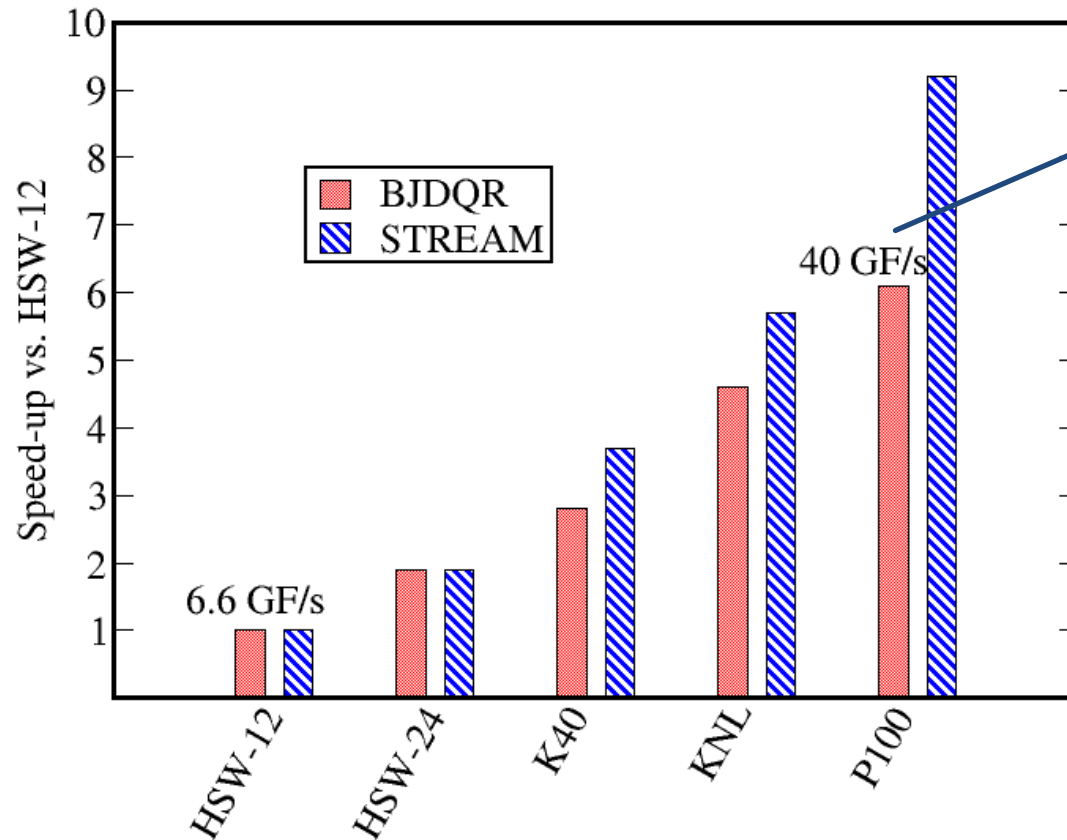
C.L. Alappat, Implementation and Performance Engineering of the Kaczmarz Method for Parallel Systems, Master Thesis

Hardware efficient (eigen) solvers

Performance Portability with PHIST+GHOST



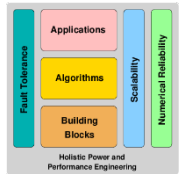
- Find 20 left-most eigenpairs of a spin-chain matrix ($N \approx 2.7M$)
- Block Jacobi-Davidson QR method with MINRES 'preconditioner' (BJQR)
- Hardware bottleneck: main memory bandwidth(?)



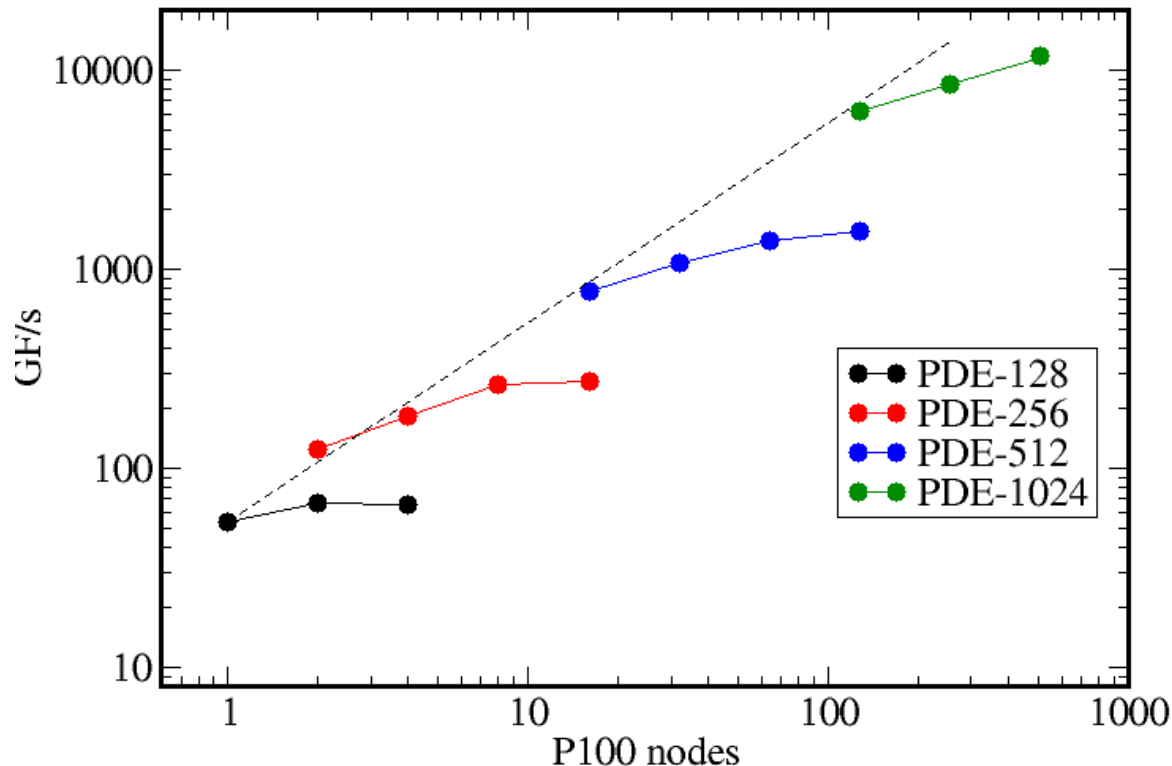
Room for improvement?!

Hardware efficient (eigen) solvers

Parallel Performance Portability: PHIST+GHOST



- Find 10 left-most eigenpairs of 3D non-symmetric PDE
- Block Jacobi-Davidson QR method with GMRES ‘preconditioner’ (BJQR)
- Scalability tests on Piz Daint (with P100) (2 vectors/block - limited memory)



Solvers & Methods

numerical efficiency

Projection based eigensolvers: Mixed precision

Finding the eigenpairs (λ, x) of $Ax = Bx\lambda$ inside $I_\lambda = [\underline{\lambda}, \bar{\lambda}]$

Single precision in early iterations:

a) Save energy b) predict behaviors in high precision operations

● : Convergence barred ● : Limited effect ● : Lasting effect

Choose m ($>$ number of desired eigenpairs) and $Y \in \mathbb{R}^{n \times m}$

while *not converged*

Construct $U \approx P_I Y$ (polynomial, contour integral, or moments) ●

Compute SVD of U , resize subspace ●

Orthogonalize U ● / ●

Solve reduced eigenproblem $A_U W = B_U W \Lambda$ ●

with $A_U := U^T A U$, $B_U := U^T B U$

$X := U W$

Orthogonalize against locked eigenpairs X , lock newly converged eigenpairs ● / ●

Projection based eigensolvers: Addressing SDC

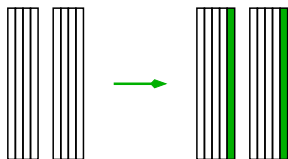
Most time-consuming part in BEAST(-P/-C):

subspace projection $U = P_{A,B} Y$

&

During (polynomial or contour-based) projection:

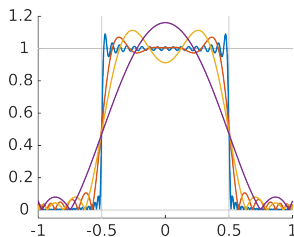
- ▶ Add checksum column(s) (linear combinations)



- ▶ $< 3\%$ overhead for 1.3M top. ins. on 160 cores, check every 100 MVMs

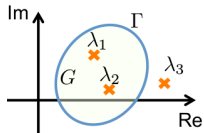
A posteriori analysis:

- ▶ Compare $\sigma_i(U^H B U)$ to max of filter function

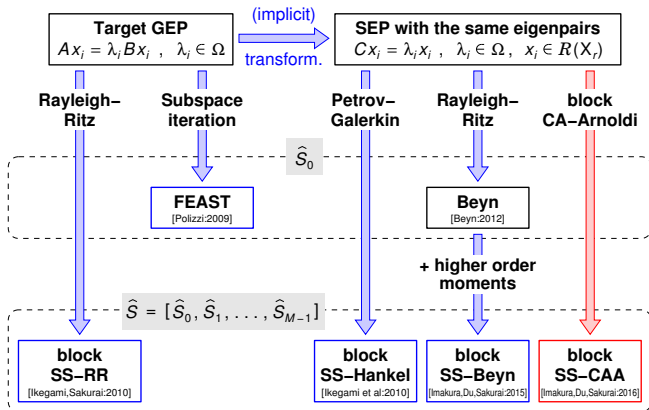


- ▶ Cheap

Block SS-CAA: A novel contour-based eigensolver



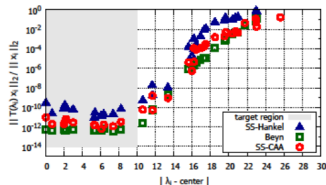
- Analyzed the relationship among typical contour integral-based eigensolvers
- Proposed novel method using communication-avoiding (CA) Arnoldi: block SS-CAA



Block SS-CAA: Results

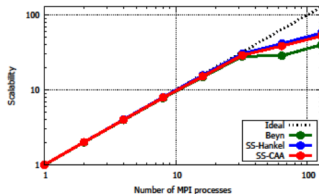
Computational accuracy:

- ▶ gen_hyper2 ($n = 1,000$)
synthetic
- ▶ NLEVP: hyperbolic
quadratic matrix
polynomial
- ▶ MATLAB2016b

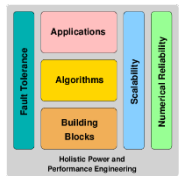
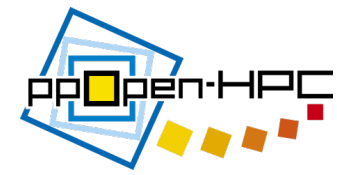


Scaling:

- ▶ railtrack2 ($n = 353,205$)
model of the vibration of
rail tracks
- ▶ NLEVP: quadratic
eigenvalue problem
- ▶ COMA@U. of Tsukuba



Preconditioner of KS method for ill-conditioned problem



Target : Linear systems
in FEAST and SSM

$$Ax = b$$

Model name	DOF	Non-zero entries per row
Graphene	128~1,000,000	4 or 13
Kohn-Sham	57,575~76,163	20~24 on average

► Properties of coefficient matrix A

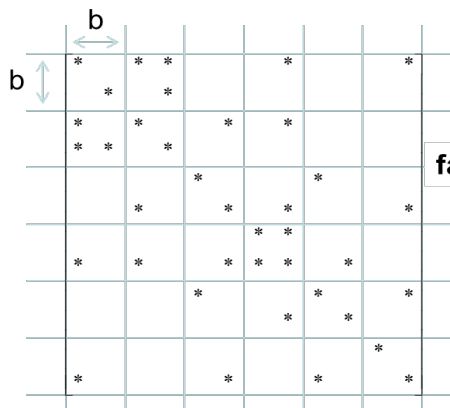
- ill-conditioned
- Indefinite
- Small diagonal entries

➡ Hard to be solved by original ICCG

Our Proposal

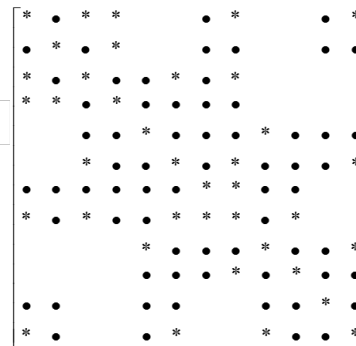
New preconditioner with regularizations

- Reg.1) Diagonal shift
- Reg.2) Blocking

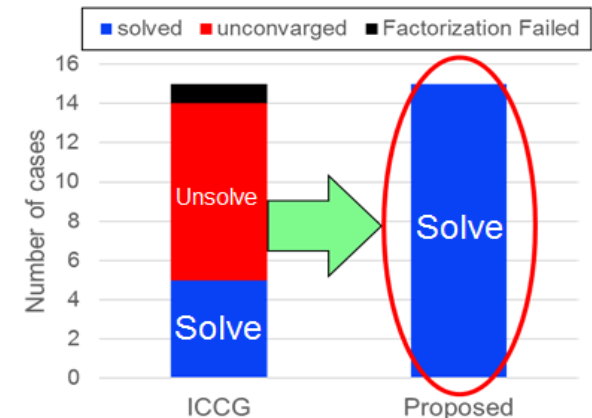
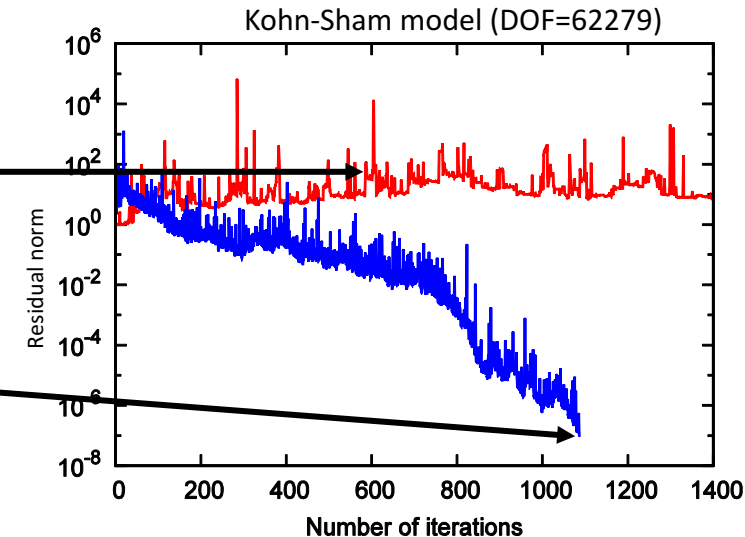


Coefficient matrix A

factorization

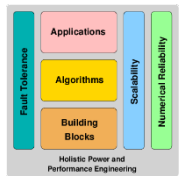


IC factorization with blocking

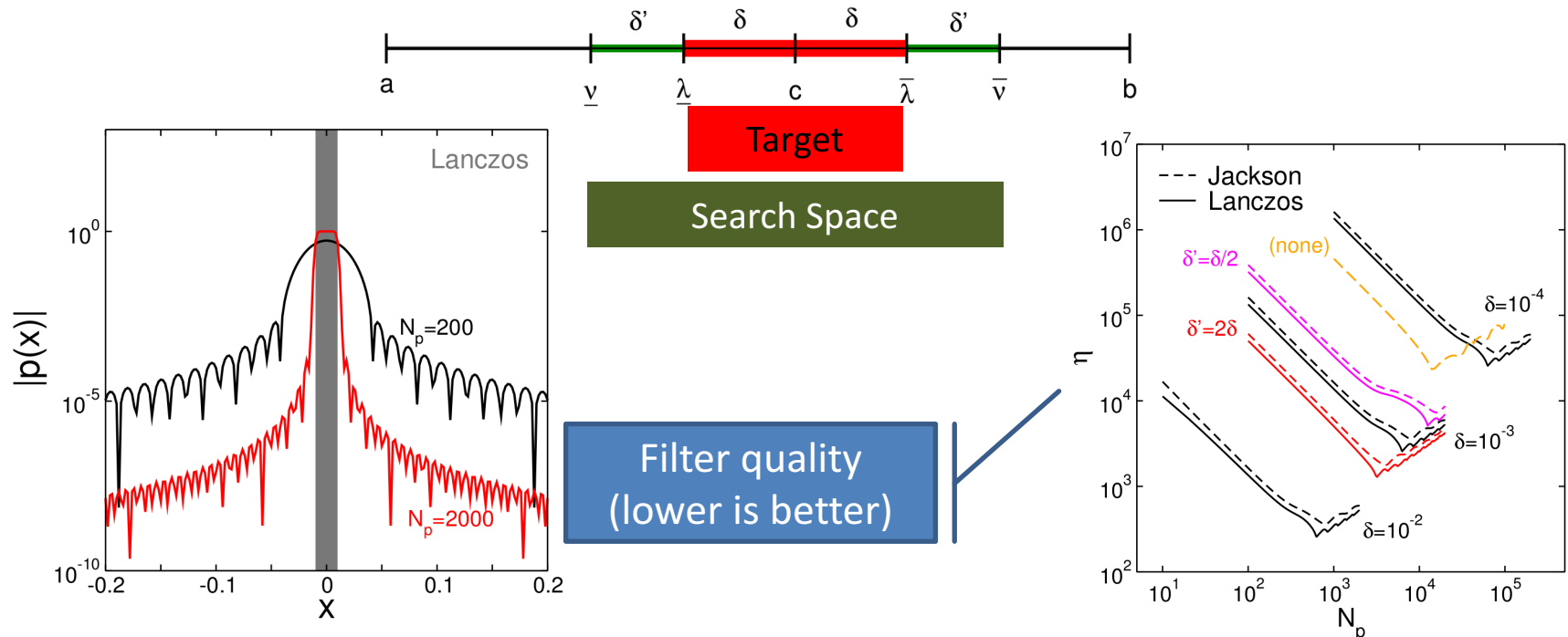


Hardware efficient solvers & applications

Chebyshev filter diagonalization: Algorithm & Performance (I)

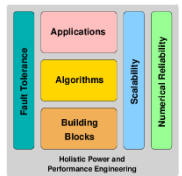


Many interior eigenvalues of large sparse (Hermitian) matrices

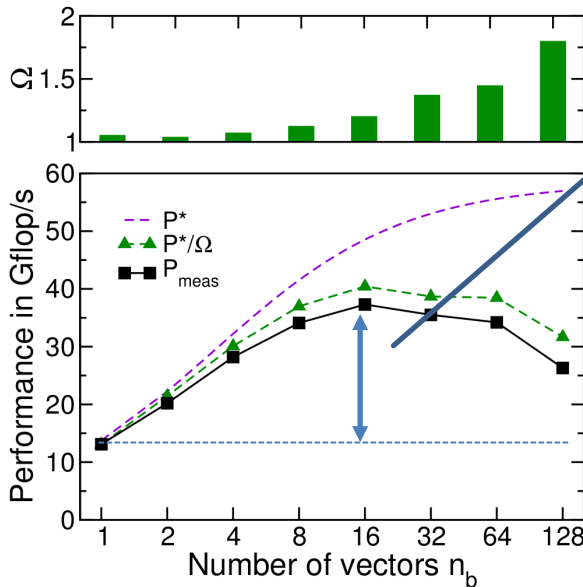


- uses only spMVM / low synch. \Rightarrow very large unstructured matrices
- convergence \leftrightarrow eigenvalue density \Rightarrow large number of spMVMs
- clue to convergence: search vectors \gg target vectors \Rightarrow spMMVM
- \Rightarrow massive vector blocking

Chebyshev filter diagonalization: Algorithm & Performance (II)



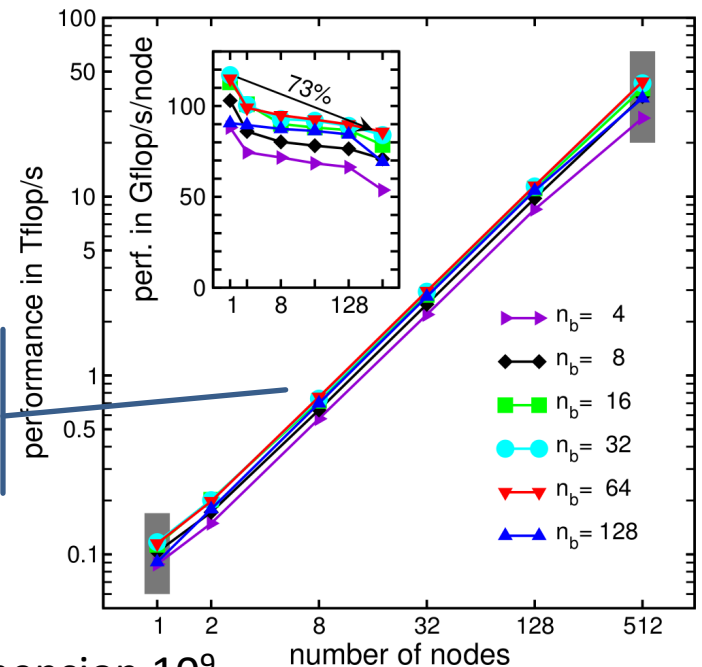
HSW-socket performance



Vector blocking
3x performance gain

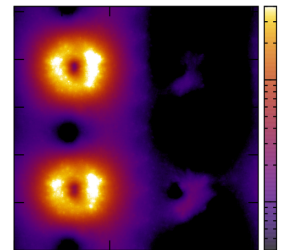
Weak scaling for
topological insulator
testcase

SuperMUC-2 performance



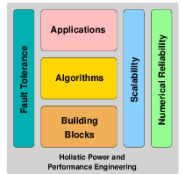
Compute 200 interior eigenpairs of a matrix of dimension 10^9
(10 hrs on 512 nodes)

Matrix	Nodes	D	$[\lambda : \bar{\lambda}]_{\text{rel}}$	N_T	N_p	Runtime [hours]	Sust. perf. [Tflop/s]
topi1	32	6.71e7	7.27e-3	148	2159	3.2 (83%)	2.96
topi2	128		3.64e-3	148	4319	4.9 (88%)	11.5
topi3	512	1.07e9	1.82e-3	148	8639	10.1 (90%)	43.9
graphene1	128		4.84e-4	104	32463	10.8 (98%)	4.6
graphene2	512	1.07e9	2.42e-4	104	64926	16.4 (99%)	18.2



Pieper, Kreutzer, Alvermann, Galgon, Fehske, Hager, Lang, Wellein, J. Comp. Phys. 325, 226 (2016)

ESSEX@PASC2017 – 26 – 28 June, 2017



**COMPUTING BULKS OF INNER EIGENPAIRS OF LARGE SPARSE MATRICES:
FROM APPLICATIONS AND ALGORITHMS TO PERFORMANCE AND
SOFTWARE ENGINEERING (I+II)**

- Takeo Hoshi (Tottori University, Japan)
- Tetsuya Sakurai (University of Tsukuba, Japan)
- Yousef Saad (University of Minnesota, USA)
- Andreas Alvermann (Universität Greifswald, Germany)
- Kengo Nakajima (The University of Tokyo, Japan)
- Hartwig Anzt (University of Tennessee, USA)
- Jonas Thies (German Aerospace Center, Germany)
- Mike Heroux (Sandia National Laboratories, USA)

<https://pasc17.pasc-conference.org/program/minisymposia/>

ESSEX II - Equipping Sparse Solvers for Exascale